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Supersonic Turbines in Space Power Applications

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Nomenclature

C_f = skin-friction coefficient
c_p = specific heat at constant pressure, kcal/(kg-°K)
g = acceleration of gravity, m/s^2
J = mechanical equivalent of heat = 427 mkg/kcal
k = thermal conductivity, cal/(cm-s-°K)
$\mathfrak{M}$ = gas-molecular weight
p = pressure, kg/cm²
Pr = Prandtl number, $c_p\mu/k$
q = heat-transfer rate per unit area, kcal/($m^2 - s$)
$\mathfrak{R}$ = universal gas constant, kcal/(kmole-°K)
r = recovery factor
r_c = compressor pressure ratio
St = Stanton number
T = absolute temperature, °K
u = blade peripheral speed, m/s
V = absolute velocity, m/s
W = relative velocity, m/s
α = angle between u and V (less than 90°)
γ = isentropic exponent
μ = viscosity coefficient, poise
ρ = density, kg/m^3

Subscripts

0 = stagnation
1 = absolute
2 = relative
3 = compressor inlet
aw = adiabatic wall
e = boundary-layer edge
w = wall

PEAK temperatures in both Rankine and Brayton cycles for energy conversion in space are limited by the heat-source characteristics, as well as by the peak temperature at which highly loaded components can operate safely. The use of supersonic turbines represents a means to lower the actual operating temperature of the turbine, which is the most critical component of high-temperature turbo-converters.

Table 1 Some properties of working fluids for space power applications

Gas or vapor	γ	Pr
Argon	1.668	0.662[a]
Krypton	1.680	0.683[b]
Xenon	1.660	0.683[b]
Xe-He Mixture[c]	1.667	0.203[d]
Methane	1.310	0.210[a]
Ethane	1.220	0.240[a]
Na, K, Cs, Hg	1.667	0.683[b]

[a] At 1500°K.
[b] Estimated[6] from $Pr = 4\gamma/(9\gamma - 5)$.
[c] 50.6% Xenon–49.4% Helium mixture; $\mathfrak{M}$ = 68.5.
[d] At 288°K.

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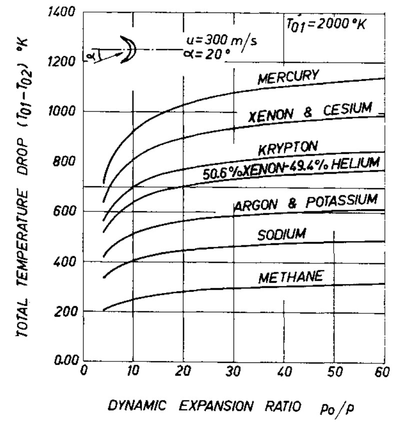

Fig. 1 Total temperature drop passing from stationary nozzles to turbine-rotating channels.

The basic characteristics of supersonic flow in turbine distributors and rotating stages are as follows: Assume that a perfect gas with constant specific heats expands isentropically from stagnation conditions characterized by a temperature T_{01} and a pressure p_0 to a pressure p. The exit velocity of the flow is given by

$$V = \left\{\frac{2\gamma gJ\mathfrak{R}}{(\gamma - 1)\mathfrak{M}}\, T_{01}\left[1 - \left(\frac{p}{p_0}\right)^{\gamma - 1/\gamma}\right]\right\}^{1/2} \quad (1)$$

The energy equation allows the calculation of the static temperature T

$$T = T_{01} - V^2/2gJc_p \quad (2)$$

where c_p is the specific heat per unit weight, another expression of which is

$$c_p = \gamma\mathfrak{R}/\mathfrak{M}(\gamma - 1) \quad (3)$$

In a turbine-rotating channel, which the flow enters at a relative velocity W, the energy equation is

$$T_{02} = T + W^2/2gJc_p \quad (4)$$

where T_{02} is the stagnation temperature in a rotating frame of reference.

The relative velocity is given by

$$W^2 = V^2 + u^2 - 2uV \cos\alpha \quad (5)$$

in which u is the turbine peripheral speed, and α is the angle, less 90°, between u and V.

Combining Eqs. (2–5), one obtains the stagnation-temperature drop, passing from stationary to rotating channels

$$T_{01} - T_{02} = \frac{2uV\cos\alpha - u^2}{2\gamma gJ\mathfrak{R}/\mathfrak{M}(\gamma - 1)} \quad (6)$$

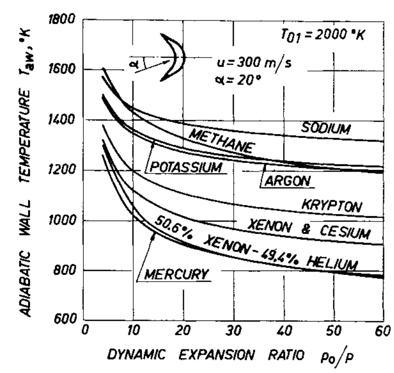

Fig. 2 Recovery temperature of adiabatic rotating blades vs stationary nozzles pressure ratio.

If V is sufficiently high (supersonic flow), and c_p sufficiently low, this temperature drop may become very significant. Assume now that the temperature recovery factor on the walls of the rotating channels is equal to that of a flat plate (this seems acceptable as a first approximation if the turning in the channel is very gradual); then

$$T_{aw} = T + r(T_{02} - T) \tag{7}$$

In a laminar boundary layer the recovery factor is given by

$$r \cong Pr^{1/2} \qquad Pr = c_p\mu/k \tag{8}$$

Let us analyze the characteristics of a number of working fluids proposed to date for space dynamic conversion systems, the most significant characteristics of which are listed in Table 1, which was compiled from Refs. 1–6; methane and ethane were added to the list, because of their very low Prandtl numbers in the range of temperatures in which we are interested.

Equations (6) and (7) are solved with these data and plotted in Figs. 1 and 2, respectively. It may be noted that for the rare gases, the assumption of constant γ and Pr is in very good agreement with experiments.

Before analyzing the results obtained so far, let us discuss another problem particularly important in Brayton-cycle turbo-generators: heat exchangers and radiators weight minimization. Power unit configurations, including heat-source heat exchanger, heat-sink heat exchanger, recuperator, and radiator, have been proposed.[5]

The favorable effect of low Prandtl number on heat-transfer rates is qualitatively shown through the Reynolds analogy, which allows one to deduce heat-transfer rates from knowledge of skin friction.

The heat transferred per unit area and per unit time is

$$q = St\rho_e V_e c_p(T_w - T_{aw}) \tag{9}$$

$$St = C_f Pr^{-2/3}/2 \tag{10}$$

where St is the Stanton number, ρ_e, V_e are the density and the velocity at the boundary-layer edge, respectively, T_w is the actual wall temperature, and C_f is the skin-friction coefficient. Equation (10) holds both for laminar and turbulent boundary layers. The foregoing results are not as simple as they first may appear, the skin-friction coefficient being dependent upon Reynolds number, Mach number, wall, and freestream temperature, and, finally, on the relationship between viscosity and temperature.[7]

The trend shown in Eqs. (9) and (10) is confirmed by detailed computations, in one of which the surface of a tabular counterflow recuperator was found to be given by the expression[5]

$$S = \lambda Pr\mathfrak{M}^{1/2}(1 + r_c^{-1/2})^2(\gamma - 1)/\gamma p_3 \tag{11}$$

where λ is a coefficient independent of fluid properties, p_3 is the pressure at the compressor inlet, and r_c is the compressor pressure ratio.

According to Eq. (11), the best performance with respect to the reduction of the recuperator weight appears to be given by methane, due to its low Pr at temperatures of practical interest and to its low $\mathfrak{M}$.

If experimental results confirm the possibility of raising the maximum gas or vapor temperature with respect to current values, this will result in improved turbo-converters efficiency and reduced radiator weight.

Figure 3 shows some possible future configurations of space dynamic power systems, employing both the Rankine and the Brayton cycles.

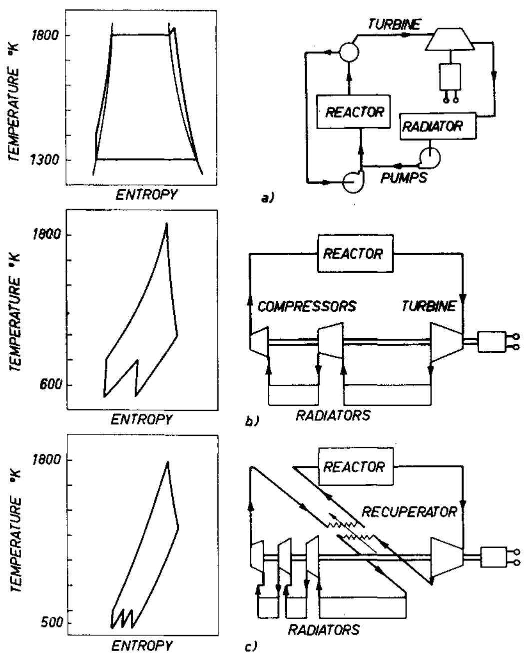

Fig. 3 Some possible configurations of turbo-converters cycles, qualified for space applications.

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Flame Spread on Solid Propellant

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Nomenclature

- c = solid heat capacity, cal/g°C
- f = surface heat flux; f_0, heat flux at edge of burning zone, cal/cm²-sec
- F = function defined by Eq. (2), determined by Eq. (1)
- k = thermal conductivity, cal/cm-sec-°C
- p = pressure, atm
- s = intermediate time variable, sec
- t = time: t_0, ignition time at $x = 0$; t_i, ignition time at position x, with time zero set at the start of gas flow; sec
- T = surface temperature rise above T_0: T_0, initial slab temperature; T_i, value of T at ignition; °C

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